Effective Chiral Theory of Mesons

Bing An Li
Department of Physics and Astronomy
University of Kentucky
Lexington, KY 40506, USA

Introduction

Before the advent of QCD the meson physics has been studied extensively since 60's.

1. Chiral symmetry, Current algebra, PCAC.
2. Vector Meson Dominance(VMD).
3. Goldstone theorem.
4. Weinberg sum rules.
5. KSFR sum rule.
6. $\pi\pi$ scattering.
7. $\pi^0 \to \gamma\gamma$ and triangle anomaly.
8. Wess-Zumino-Witten Lagrangain.
9. Chiral perturbation theory.
10. Quark model leads to a energy scale: constituent quark mass.

How can we have a theory to unify all these studies? This theory should be QCD inspired, self-consistent, and phenomenologically successful. We have proposed a theory (B.A. Li, Phys. Rev. **D** **52**5165-5183, 5184-5193(1995)).

Simulations of the meson fields

The key point is how to bosonize QCD. As a matter of fact, in the bosonization of 1+1 filed theory

$$\frac{1}{\sqrt{\pi}}\partial_\mu\phi = \bar{\psi}\gamma_5\gamma_\mu\psi.$$

Following this idea, we propose to simulate the meson fields by quark operators. For example,

$$\rho^i_\mu = -\frac{1}{g_\rho m^2_\rho}\bar{\psi}\tau_i\gamma_\mu\psi.$$

Model independent test

Using PCAC, current algebra, and this expression, in the limit t of $p_\pi \to 0$, we obtain

$$\frac{1}{2}f_{\rho\pi\pi}g_\rho = 1,$$

$$A = \frac{2}{f_\pi}(1-\frac{1}{2\pi^2 g^2})^{-\frac{1}{2}}(m^2_a - m^2_\rho).$$

The octet pseudoscalars are Goldstone bosons and we treat pseudoscalar mesons differently from others.

realization of quark operator expressions of meson fields

Using $U(2)_L \times U(2)_R$ chiral symmetry and the minimum coupling principle, the Lagrangian is constructed as

$$\mathcal{L} = \bar{\psi}(x)(i\gamma\cdot\partial + \gamma\cdot v + \gamma\cdot a\gamma_5 - mu(x))\psi(x) - \bar{\psi(x)}M\psi(x)$$
$$+\frac{1}{2}m^2_0(\rho^\mu_i\rho_{\mu i} + \omega^\mu\omega_\mu + a^\mu_i a_{\mu i} + f^\mu f_\mu) \quad (1)$$

where $a_\mu = \tau_i a^i_\mu + f_\mu$, $v_\mu = \tau_i \rho^i_\mu + \omega_\mu$, and $u = exp\{i\gamma_5(\tau_i\pi_i + \eta)\}$. u can be written as

$$u = \frac{1}{2}(1+\gamma_5)U + \frac{1}{2}(1+\gamma_5)U^\dagger, \tag{2}$$

where $U = exp\{i(\tau_i\pi_i + \eta)\}$. Since mesons are bound states solutions of QCD they are not independent degrees of freedom. Therefore, there are no kinetic terms for meson fields. The kinetic terms of meson fields are generated from quark loops. The scheme of nonlinear σ model is used to introduce the pseudoscalar meson fields. Using the least action principle, we obtain

$$\begin{aligned}
\frac{\Pi_i}{\sigma} &= i(\bar\psi\tau_i\gamma_5\psi + ix\bar\psi\tau_i\psi)/(\bar\psi\psi + ix\bar\psi\gamma_5\psi), \\
x &= (i\bar\psi\gamma_5\psi - \frac{\Pi_i}{\sigma}\bar\psi\tau_i\psi)/(\bar\psi\psi + i\frac{\Pi_i}{\sigma}\bar\psi\tau_i\gamma_5\psi), \\
\rho^i_\mu &= -\frac{1}{m_0^2}\bar\psi\tau_i\gamma_\mu\psi, \quad a^i_\mu = -\frac{1}{m_0^2}\bar\psi\tau_i\gamma_\mu\gamma_5\psi, \\
\omega_\mu &= -\frac{1}{m_0^2}\bar\psi\gamma_\mu\psi, \quad f\mu = -\frac{1}{m_0^2}\bar\psi\gamma_\mu\gamma_5\psi,
\end{aligned} \tag{3}$$

where $u = e^{i\eta\gamma_5}(\sigma + i\gamma_5\tau\cdot\Pi)$, $\sigma = \sqrt{1-\Pi^2}$, and $x = tan\eta$. The pseudoscalar fields have very complicated quark structures.

Substituting theses expressions into the Lagrangian it becomes a Lagrangian of quarks.

Using the method of path integral to integrate out the quark fields, the effective Lagrangian of mesons(indicated by M) is obtained

$$\mathcal{L}^M_E = logdet\mathcal{D}, \tag{4}$$

where

$$\mathcal{D} = \gamma\cdot\partial - i\gamma\cdot v - i\gamma\cdot a\gamma_5 + mu. \tag{5}$$

The Wess-Zumino-Witten Lagrangian with spin-1 fields is obtained from the Lagrangian and it is the leading term of $\mathcal{L}_{IM}$ in derivative expansion.

All the vertices of the meson physical processes of normal parity can be found from $\mathcal{L}_{RE}$ and all the vertices of abnormal parity can be derived from $\mathcal{L}_{IM}$.

Defining physical meson fields

The physical meson fields can be defined in the following way that makes the corresponding kinetic terms in the standard form:

$$\pi \longrightarrow \frac{2}{f_\pi}\pi, \quad \eta \longrightarrow \frac{2}{f_\eta}\eta,$$
$$\rho \longrightarrow \frac{1}{g}\rho, \quad \omega \longrightarrow \frac{1}{g}\omega, \tag{6}$$
$$c = \frac{f_\pi^2}{2gm_\rho^2}. \tag{7}$$
$$a_\mu^i \longrightarrow \frac{1}{g}(1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}} a_\mu^i - \frac{c}{g}\partial_\mu \pi^i. \tag{8}$$
$$f_\mu \longrightarrow \frac{1}{g}(1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}} f_\mu - \frac{c}{g}\partial_\mu \eta_0. \tag{9}$$

In this theory the parameters are:

m_u, m_d, m_s, g, m.

g is an universal coupling constant. Input f_π, we determine $m = 300MeV$ and choose $g = 0.35$.

Large N_c expansion

Due to the quark loops N_c is included in this theory. All tree diagrams are of order $O(N_c)$, hence they are leading contributions. A diagram with loops is at higher order in large N_c expansion. For instance, a diagram of one loop with two internal lines is of order $O(1)$. Therefore, the large N_c expansion is the loop expansion in this theory.

Cut-off

This theory is an effective theory and it is not renormalizable, as mentioned above. Therefore, a cut-off of momentum has to be introduced. Using a cut-off instead dimensional regularization, we have

$$\frac{N_c}{(4\pi)^2}\{log(1+\frac{\Lambda^2}{m^2}) + \frac{1}{1+\frac{\Lambda^2}{m^2}} - 1\} = \frac{1}{16}\frac{F^2}{m^2} = \frac{3}{8}g^2. \tag{10}$$

Using the values of m and g, we obtain

$$\Lambda = 1.6GeV. \tag{11}$$

The masses of all mesons in this theory are below the cut-off.

Dynamical chiral symmetry breaking

The quark condensate is defined as

$$< 0|\bar{\psi}(x)\psi(x)|0 >= -m^3 \frac{N_C}{8\pi^2}\{\frac{\Lambda^2}{m^2} - log(1 + \frac{\Lambda^2}{m^2})\}. \tag{12}$$

In nonperturbative QCD a **dynamical quark mass**(constituent quark mass)is introduced

$$< 0|\bar{\psi}(x)\psi(x)|0 >= -m^3 \frac{3}{4\pi\alpha_s}. \tag{13}$$

Therefore, the parameter m of this theory is just the dynamical quark mass in nonperturbative QCD and

$$\alpha_s(1.6GeV) = \frac{4\pi}{\frac{\Lambda^2}{m^2} - log(1 + \frac{\Lambda^2}{m^2})} = 0.50. \tag{14}$$

There is dynamical chiral symmetry breaking in this theory. On the other hand, the PCAC is derived

$$\partial^\mu \bar{\psi}\tau_i\gamma_\mu\gamma_5\psi = -m_\pi^2 f_\pi \pi_i.$$

Masses

To the leading order in quark mass expansion, the masses of the octet pseudoscalar mesons are derived(Gell-Mann, Oakes, Renner formulas)

$$\begin{aligned}
m_\pi^2 &= -\frac{2}{f_\pi^2}(m_u + m_d) < 0|\bar{\psi}\psi|0 >, \\
m_{K^+}^2 &= -\frac{2}{f_\pi^2}(m_u + m_s) < 0|\bar{\psi}\psi|0 >, \\
m_{K^0}^2 &= -\frac{2}{f_\pi^2}(m_d + m_s) < 0|\bar{\psi}\psi|0 >, \\
m_\eta^2 &= -\frac{2}{3f_\pi^2}(m_u + m_d + 4m_s) < 0|\bar{\psi}\psi|0 > .
\end{aligned} \tag{15}$$

$$m_u = 4.64MeV, \quad m_d = 8.16MeV, \quad m_s = 159MeV. \tag{16}$$

The KSFR sum rule

$$g_\rho = \frac{1}{2} f_{\rho\pi\pi} f_\pi^2 \tag{17}$$

is derived by using this theory and it can be taken as the equation used to determine m_ρ.

$$f_{\rho\pi\pi} = \frac{2}{g}, \tag{18}$$

$$g_\rho = \frac{1}{2} g m_\rho^2. \tag{19}$$

We obtain

$$m_\rho^2 = 6m^2. \tag{20}$$

In the limit of $m_q = 0$, the mass of ρ meson originates from dynamical chiral symmetry breaking.

$$m_\rho = 0.751 GeV. \tag{21}$$

In the limit of $m_q = 0$, the masses of the four low lying vector mesons originate from dynamical chiral symmetry breaking.

$$f_\pi^2 = 3g^2 m^2. \tag{22}$$

The pion decay constant is the result of dynamical chiral symmetry breaking too. Therefore, in the limit of $m_q \to 0$, the decay constants of the octet pseudoscalar mesons originate from dynamical chiral symmetry breaking.

From the spontanous chiral symmetry breaking we obtain

$$(1 - \frac{1}{2\pi^2 g^2}) m_a^2 = 6m^2 + m_\rho^2. \tag{23}$$

$$(1 - \frac{1}{2\pi^2 g^2}) m_f^2 = 6m^2 + m_\omega^2. \tag{24}$$

Adding the strangeness to the Lagrangian, we obtain

$$(1 - \frac{1}{2\pi^2 g^2}) m_{K_1(1400)}^2 = 6m^2 + m_{K^*(892)}^2$$
$$(1 - \frac{1}{2\pi^2 g^2}) m_{f_1(1510)}^2 = 6m^2 + m_\phi^2. \tag{25}$$

Summary of the results

1. Vector meson dominance(VMD)

$$
\begin{aligned}
&\frac{e}{f_\rho}\{-\frac{1}{2}F^{\mu\nu}(\partial_\mu\rho^0_\nu-\partial_\nu\rho^0_\mu)+A^\mu j^0_\mu\},\\
&\frac{e}{f_\omega}\{-\frac{1}{2}F^{\mu\nu}(\partial_\mu\omega_\nu-\partial_\nu\omega_\mu)+A^\mu j^\omega_\mu\},\\
&\frac{e}{f_\phi}\{-\frac{1}{2}F^{\mu\nu}(\partial_\mu\phi_\nu-\partial_\nu\phi_\mu)+A^\mu j^\phi_\mu\},
\end{aligned}
\tag{26}
$$

   where $f_\rho = \frac{1}{2}g$, $f_\omega = \frac{1}{6}g$, and $f_\phi = -\frac{1}{3\sqrt{2}}g$.

2. Weinberg's first sum rule is satisfied analytically.

3. The scattering lengths and slopes of $\pi\pi$ scattering obtained by Weinberg are revealed from this theory. It is found that ρ meson exchange dominates $\pi\pi$ scattering.

4. In the chiral limits, two coefficients of chiral perturbation theory are determined and in good agreement with CPT.

5. The amplitude of $\pi^0 \to \gamma\gamma$ obtained in this theory is the same as the triangle anomaly.

6. The theory provides a unified description of meson physics at low energies. In this unified description universal coupling in all the physical processes has been found and the inputs are the cut-off Λ, m(related to quark condensate), and the quark masses. The theory is self-consistent and phenomenologically successful.

Table 1: Summary of the results

	Experimental	Theoretical
f_π	186MeV	input
g		0.35 input
m_ω	781.94±0.12MeV	770MeV
m_a	1230±40MeV	1389 MeV
m_{f_1}	1282 ± 5MeV	1389 MeV
π form factor	consistent with ρ pole	ρ pole
radius of π	0.663±0.023fm	0.63 fm
$g_{\rho\gamma}$	$0.116(1 \pm 0.05)\ GeV^2$	$0.104\ GeV^2$
$g_{\omega\gamma}$	$0.0359(1 \pm 0.03)\ GeV^2$	$0.0357\ GeV^2$
$\Gamma(\rho \to \pi\pi)$	151.2 ± 1.2 MeV	135. MeV
$\Gamma(\omega \to \pi\pi)$	$0.186(1 \pm 0.15)$MeV	0.136MeV
$\Gamma(a_1 \to \rho\pi)$	~400 MeV	325 MeV
$\Gamma(a_1 \to \gamma\pi)$	(640±246)keV	252keV
$\frac{d}{s}(a_1 \to \rho\pi)$	-0.11 ± 0.02	-0.097
$\Gamma(\tau \to a_1\nu)$	$(2.42 \pm 0.76)10^{-13}$ GeV	1.56×10^{-13}GeV
$\Gamma(\tau \to \rho\nu)$	$(0.495 \pm 0.023)10^{-12}$GeV	4.84×10^{-13}GeV
$\Gamma(\pi^0 \to \gamma\gamma)$	$7.74(1 \pm 0.072)$eV	7.64eV
a(form factor of $\pi^0 \to \gamma\gamma$)	0.032± 0.004	0.03
$\Gamma(\omega \to \pi\gamma)$	717(1±0.07)keV	724 keV
$\Gamma(\rho \to \pi\gamma)$	68.2(1±0.12)keV	76.2keV
$\Gamma(\omega \to \pi\pi\pi)$	7.43(1± 0.02)MeV	5 MeV
$\Gamma(f_1 \to \rho\pi\pi)$	6.96(1±0.33)MeV	6.01MeV
$B(f_1 \to \eta\pi\pi)$	$(10^{+7}_{-6})\%$	1.15×10^{-3}
$\Gamma(f_1 \to \gamma\pi\pi)$		18.5keV
$B(\rho \to \gamma\eta)$	$(3.8 \pm 0.7) \times 10^{-4}$	3.04×10^{-4}
$B(\omega \to \gamma\eta)$	$(8.3 \pm 2.1) \times 10^{-4}$	6.96×10^{-4}

Table 2: Table II

	Experimental	Theoretical
m_{K_1}	1402±7MeV	1510 MeV
$m_{f_1(1510)}$	1512±4MeV	1640 MeV
$g_{\phi\gamma}$	$0.081(1 \pm 0.05)\ GeV^2$	$0.086\ GeV^2$
$< r^2 >_{K^\pm}$	$0.34 \pm 0.05 fm^2$	$0.33 fm^2$
$< r^2 >_{K^0}$	$0.054 \pm 0.026 fm^2$[11]	$0.0582 fm^2$
$\lambda_+(K^+_{l3})$	0.0286 ± 0.0022	0.0239
$\xi(K^+_{l3})$	-0.35 ± 0.15	-0.284
$\lambda_+(K^0_{l3})$	0.03 ± 0.0016	0.0245
$\xi(K^0_{l3})$	-0.11 ± 0.09	-0.287
$\Gamma(K^+_{e3})$	$0.256(1 \pm 0.015)10^{-17}$GeV	0.233×10^{-17}GeV
$\Gamma(K^0_{e3})$	$0.493(1 \pm 0.016)10^{-17}$GeV	0.483×10^{-17}GeV
$B(\tau \to K^*(892)\nu)$	$(1.45 \pm 0.18)\%$	1.46%
$\Gamma(\tau \to K_1(1400)\nu)$		0.373%
$\Gamma(\phi \to K^0\bar{K}^0)$	$1.52(1 \pm 0.03)$ MeV	1.11MeV
$\Gamma(\phi \to K^+K^-)$	$2.18(1 \pm 0.03)$MeV	1.7MeV
$\Gamma(K^*(892) \to K\pi)$	49.8 ± 0.8MeV	39.4 MeV
$\Gamma(K^{+*} \to K^+\gamma)$	$50.3(1 \pm 0.11)$keV	43.5keV
$\Gamma(K^{0*} \to K^0\gamma)$	$116.2(1 \pm 0.10)$keV	175.4keV
$B(K^*(892) \to K\pi\pi)$	0.53×10^{-4}	$< 7 \times 10^{-4}$
$f_1(1510) \to K^*(892)\bar{K})$	35 ± 15MeV	22.MeV
$\Gamma(K_1(1400) \to K^*(892)\pi)$	$163.6(1 \pm 0.14)$ MeV	126 MeV
$B(K_1(1400) \to K\rho)$	$(3.0 \pm 3.0)\%$	11.1%
$B(K_1(1400) \to K\omega)$	$(2.0 \pm 2.0)\%$	2.4%
$\Gamma(K_1 \to K\gamma)$		440keV

$\Gamma(\eta' \to \eta\pi^+\pi^-)$	$87.8(\pm 0.12)$keV	85.7keV
$\Gamma(\eta' \to \eta\pi^0\pi^0)$	$41.8(\pm 0.11)$keV	48.6keV
$\Gamma(\eta \to \gamma\gamma)$	$0.466(1 \pm 0.11)$keV	0.619keV
$\Gamma(\phi \to \eta\gamma)$	$56.7(1 \pm 0.06)$keV	91.4keV
$\Gamma(\rho \to \eta\gamma)$	$57.5(1 \pm 0.19)$keV	61.4keV
$\Gamma(\omega \to \eta\gamma)$	$7.0(1 \pm 0.26)$keV	7.84keV
$\Gamma(\eta' \to \gamma\gamma)$	$4.26(1 \pm 0.14)$keV	4.88keV
$\Gamma(\eta' \to \rho\gamma)$	$60.7(1 \pm 0.12)$keV	63.0keV
$\Gamma(\eta' \to \omega\gamma)$	$6.07(1 \pm 0.18)$keV	5.86keV

Conclusions

1. The masses of the octet pseudoscalar mesons are proportional to current quark masses. In chiral limit, they are massless.

2. The masses of vector mesons originate from dynamical chiral symmetry breaking.

3. The mass differences of vector and axial-vector mesons are caused by spontaneous chiral symmetry breaking.

4. The cut-off Λ(1.6GeV) is at the border of nonperturbative QCD and perturbative QCD. Perturbative QCD make corrections.

5. In the case of two flavors theoretical results agree with data within about 10% and if strange quark is involved, in the chiral limit the worst case is $\Gamma(\phi \to K\bar{K})$ which is less than data by 30%. The strange quark mass correction should be taken into consideration.

6. The Lagrangian is not closed. Other mesons and glueballs, in principle, could be included.

7. The parameter m and Λ should be determined by full QCD.